Math 266.3 Midterm — Oct 24—2003

1. Determine an equation of the quadratic polynomial which passes through the points $(-1, 1)$, $(1, -5)$, and $(2, -2)$. $n=3$ $Ax^2+Bx+C=0$

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2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, then $6=1+1-3-1-2$

~~(a)~~ find the trace tr(A) of A = $2\cdot 3\cdot 1 = 6$

~~(b)~~ find the determinant of A, det(A), by using the cofactor expansion in the third column.

~~(c)~~ find A^2

~~(d)~~ use the method of Gauss-Jordan elimination to find A^{-1}. Use an augmented matrix, and indicate which row operations are being performed at each step:

~~(e)~~ encode the message "UPTAKE" using the coding matrix A.

~~(f)~~ the message 13,5,28,16 was encoded using the coding matrix $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Decode it.

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3. (a) Find the angle between the vectors $(1, 2, -2)$ and $(3, -1, 1)$.

~~(b)~~ Find and sketch the image of the triangle with vertices $(-1, 1)$, $(1,1)$, $(0,2)$ under the mapping $T(x, y) = (x + y + 1, y - 2)$

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4. Prove that a transformation defined by a 2×2 nonsingular matrix always maps straight lines into straight lines.

THE END

University of Saskatchewan
Department of Mathematics & Statistics

Instructor: M. Marshall | MATH 266.3 (02)
Time 50 minutes | Term Test 3 | April 3, 2002

Marks:

[6] **1.** Let A be an $m \times n$ matrix.

(a) Define the row space of A, the column space of A and the nullspace of A.

(b) Define the rank of A and the nullity of A and describe the relationship between the two.

[6] **2.** Let $A = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 3 & 4 & 5 \end{pmatrix}$.

(a) Determine the reduced row echelon form U of the matrix A.

(b) Describe the relationship between the columns of A. In particular, determine a basis for the column space of A.

(c) Determine the nullspace of A and a basis for the nullspace of A.

[7] **3.** Let V and W be vector spaces.

(a) Define what it means to say that a function $L : V \to W$ is a linear transformation.

(b) Define the kernel of a linear transformation $L : V \to W$.

(c) Define $L(S)$ where $L : V \to W$ is a linear transformation and S is a subspace of V.

(d) Prove that if $L : V \to W$ is a linear transformation and S is a subspace of V then $L(S)$ is a subspace of W.

[5] **4.** Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $L(x) = (x_1, x_1 - x_2)^T$.

(a) Determine the matrix of L with respect to the standard ordered basis $[e_1, e_2]$ of $\mathbb{R}^2$.

(b) Determine the matrix of L with respect to the ordered basis $[v_1, v_2]$ of $\mathbb{R}^2$ where $v_1 = (1,1)^T$, $v_2 = (1,-1)^T$.

(c) Determine an invertible 2×2 matrix S such that $B = S^{-1}AS$ where A is the matrix from part (a) and B is the matrix from part (b).

[24] Total ** The End **

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

University of Saskatchewan
Department of Mathematics & Statistics

Instructor: M. Marshall | MATH 266.3 (02)
Time 50 minutes | Term Test 2 | March 6, 2002

Marks:

[3] **1.** For what values of b is the matrix $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & b & -2 \\ b & 1 & 3b \end{pmatrix}$ non-singular?

[3] **2.** Use the Wronskian Test to decide if the functions $1, x, x^2, x^3$ are linearly independent.

[6] **3.** Let $S = \{(2a+b, a, b)^T \mid a, b \in \mathbb{R}\}$.

(a) Prove that S is a subspace of the vector space $\mathbb{R}^3$.
(b) Determine a basis for S. Justify your answer.

[6] **4.** Let $\underline{v}_1 = (1, 0, -1)^T$, $\underline{v}_2 = (2, 1, 4)^T$, $\underline{v}_3 = (7, 2, 5)^T$.

(a) Express $\underline{v}_3$ as a linear combination of $\underline{v}_1$ and $\underline{v}_2$.
(b) Describe $\mathrm{Span}(\underline{v}_1, \underline{v}_2, \underline{v}_3)$ geometrically.

[6] **5.** Let A be a fixed $m \times n$ matrix.

(a) Define the nullspace $\mathrm{N}(A)$ of A.
(b) Prove that $\mathrm{N}(A)$ is a subspace of $\mathbb{R}^n$.

[24] Total

** The End **

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

University of Saskatchewan
Department of Mathematics & Statistics

Instructor: M. Marshall — MATH 266.3 (01)

Time 50 minutes — Term Test 2 — November 7, 2001

Marks:

[6] **1.** (a) State the definition of a subspace of a vector space.

(b) Verify that the set S consisting of all polynomials f of degree less than 3 which satisfy $f(2) = 0$ is a subspace of the vector space P_3.

(c) Determine a basis for the vector space S in part (b). Explain.

[6] **2.** Suppose that V is a vector space and that $\underline{v}_1, \dots, \underline{v}_k$ are elements of V. As usual, $\text{Span}(\underline{v}_1, \dots, \underline{v}_k)$ denotes the set of all linear combinations of $\underline{v}_1, \dots, \underline{v}_k$.

(a) Prove that $\text{Span}(\underline{v}_1, \dots, \underline{v}_k)$ is a subspace of V.

(b) Prove that if $\underline{v}_1, \dots, \underline{v}_k$ are linearly independent then each vector $\underline{v}$ in $\text{Span}(\underline{v}_1, \dots, \underline{v}_k)$ is expressible uniquely as a linear combination of $\underline{v}_1, \dots, \underline{v}_k$.

[6] **3.** Let $A = \begin{pmatrix} -1 & 0 & 2 \\ 2 & 1 & 1 \\ 0 & 1 & 5 \end{pmatrix}$.

(a) Determine the rank and nullity of A. Explain.

(b) Determine a basis for the column space of A. Explain.

(c) Determine a basis for the null space of A. Explain.

[6] **4.** Consider the ordered bases $[\underline{u}_1, \underline{u}_2]$ and $[\underline{v}_1, \underline{v}_2]$ of $\mathbb{R}^2$ defined by $\underline{u}_1 = (1,1)^T$, $\underline{u}_2 = (1,3)^T$, $\underline{v}_1 = (0,1)^T$, $\underline{v}_2 = (1,2)^T$.

(a) Determine the transition matrix from $[\underline{u}_1, \underline{u}_2]$ to $[\underline{v}_1, \underline{v}_2]$.

(b) Express $5\underline{u}_1 - 3\underline{u}_2$ as a linear combination of $\underline{v}_1$, $\underline{v}_2$.

[24] Total

** The End **

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX

University of Saskatchewan
Department of Mathematics & Statistics

Instructor: M. Marshall — MATH 266.3 (02)

Time 50 minutes — Term Test 1 — February 1, 2002

Marks:

[6] **1.** Consider the system of linear equations:

$$\begin{aligned} x_1 - x_2 &= 3 \\ 2x_1 - 2x_2 + x_3 + x_4 &= 8 \\ x_1 - x_2 + x_3 + x_4 &= 5 \end{aligned}$$

(a) Write a single matrix equation which is equivalent to this linear system.
(b) Determine the general solution of this linear system using the method of Gaussian elimination. Show your work.

[8] **2.** Let $B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ -1 & 3 & 5 \end{pmatrix}$.

(a) Determine elementary matrices E_1, E_2, E_3 such that $E_3E_2E_1B$ is upper triangular.
(b) Express B in the form $B = LU$ where L is lower triangular and U is upper triangular.

[4] **3.** Let $\underline{b}_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $\underline{b}_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$, $\underline{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and let $\underline{e}_1, \underline{e}_2, \underline{e}_3$ denote the standard basis of $\mathbb{R}^3$.

(a) Express $\underline{b}$ as a linear combination of $\underline{e}_1, \underline{e}_2$ and $\underline{e}_3$.
(b) Is $\underline{b}$ expressible as a linear combination of $\underline{b}_1$ and $\underline{b}_2$? Explain.

[6] **4.** Prove that if A is any $m \times n$ matrix, B is any $n \times r$ matrix, and α is any scalar, then

$$\alpha(AB) = (\alpha A)B = A(\alpha B).$$

[24] Total

** The End **

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

University of Saskatchewan

[S. Kuhlmann] MATHEMATICS 266.3

Time: 50 minutes *SECOND MIDTERM TEST* March 11, 2005

CLOSED BOOK, NO NOTES.

Show your work. Justify your answers.

1) a) Define the notion of an **invertible matrix**.

b) Prove that if A is an invertible $n \times n$ matrix, then for any each $n \times 1$ (column matrix) $\mathbf{b}$, the system of equations

$$A\mathbf{x} = \mathbf{b}$$

has exactly one solution.

c) Solve the system

$$\begin{array}{rcrcl} 3x_1 & + & 5x_2 & = & b_1 \\ x_1 & + & 2x_2 & = & b_2 \end{array}$$

by inverting the coefficient matrix.

2) a) Let A be an $n \times n$ matrix. Define **a signed elementary product from A** and the **determinant** of A.

b) Find a 3×3 triangular matrix D that satisfies that $\det(2D) = 16$.

c) Evaluate the determinant by reducing the matrix to row echelon form:

$$A = \begin{pmatrix} 4 & 2 & 6 & 2 \\ 2 & 0 & 2 & 2 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 4 & 6 \end{pmatrix}$$

3) a) Let V be a vector space and $S \subseteq V$. Define the **span of** S.

b) Prove that the span of S is a subspace of V.

c) Let $V = M_{2\times 2}$ the vector space of 2×2 matrices. Find 4 matrices that span V.

** The End **